MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

DMT5241 – MATHEMATICAL TECHNIQUES 2

(For DIT students only)

29 FEBRUARY 2020
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **2** pages with **4** questions excluded the cover page and Appendix. Key formulae are given in the Appendix.

2. Answer **ALL** questions.

3. Write your answers in the answer booklet provided.

4. All necessary working steps must be shown.

Question 1

a. Given $f(x)=4x^3-7$ and $g(x)=3x^7+1$, find the second derivative, $\frac{d^2y}{dx^2}$ if $y=f(x).g(x)$. (4 marks)

b. By using implicit differentiation, show that the first derivative of $2e^{-5y}=5x^2-11$ is $\frac{dy}{dx}=-xe^{5y}$. (2.5 marks)

c. A piece of cube metal is expanding at a rate of $6cm^3s^{-1}$ when it is being heated. How fast is the side length increasing when the side length of the cube is $5cm$? (3.5 marks)

[TOTAL 10 MARKS]

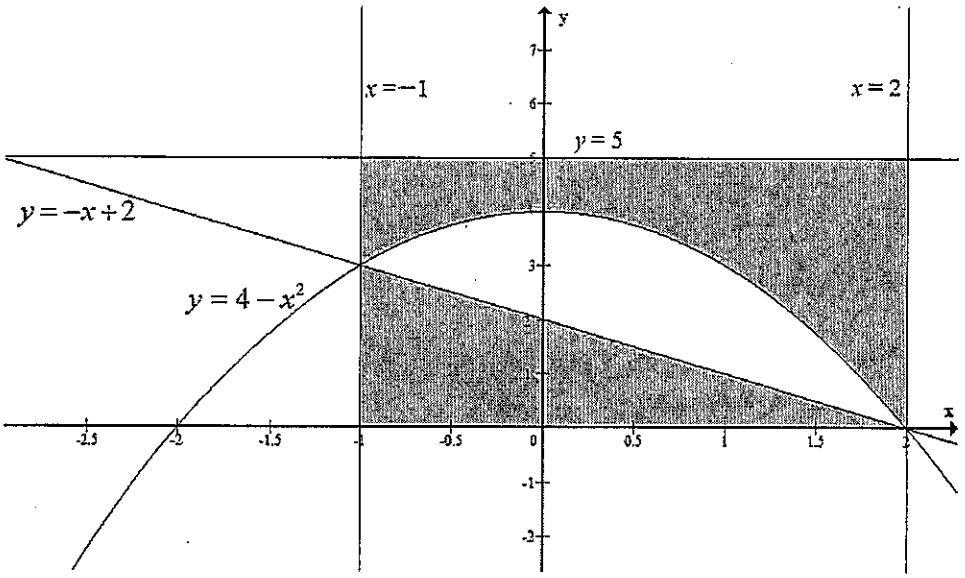
Question 2

a. Integrate $\int 4x+\frac{5}{x}-\sec^2 x dx$. (2 marks)

b. Use substitution method to integrate $\int (x+5)^3 dx$. (2.5 marks)

c. Given that $\int_a^c f(x)dx+\int_c^b f(x)dx=\int_a^b f(x)dx$ and $\int_a^b f(x)dx=-\int_b^a f(x)dx$, find $\int_0^5 f(x)dx$ if $\int_0^2 f(x)dx=10$ and $\int_5^2 f(x)dx=17$. (1 mark)

d. Find the area of the shaded region. (4.5 marks)



[TOTAL 10 MARKS]

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Question 3

a. Solve the separable differential equation, $\frac{dm}{dx} = \frac{10e^{-5x}}{6m^2 + 2m}$. (2 marks)

b. Given the differential equation, $\frac{dy}{dx} - \frac{2}{x}y = x^3 + 1$,

i. identify $p(x)$ and $q(x)$. (1 mark)

ii. determine the integrating factors, μ. (1 mark)

iii. show that the general solution of the differential equation is $y = \frac{x^4}{2} - x + cx^2$. (3 marks)

c. Find the general solution of homogeneous equation, $2y'' + 5y' - 12y = 0$. (3 marks)

[TOTAL 10 MARKS]

Question 4

a. Given that vectors, $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$.

i. Find $|\mathbf{a}|$ and $|\mathbf{b}|$. (3 marks)

ii. Compute $\mathbf{a} \bullet \mathbf{b}$. (1 mark)

iii. Find the angle between the vectors **a** and **b**. (1 mark)

b. Given a line passing through the points (3, 2, 2) and (4, –1, 5), compute

i. the parametric equations of the line. (4 marks)

ii. the symmetric equations of the line. (1 mark)

[TOTAL 10 MARKS]

End of Page

APPENDIX

Derivatives: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Differentiation Rules

General Formulae

1. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$
2. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
3. $\frac{d}{dx}(x^n) = nx^{n-1}$
4. $\frac{d}{dx}[f(u)] = \frac{dy}{du}.\frac{du}{dx}$

Exponential and Logarithmic Functions

1. $\frac{d}{dx}(e^x) = e^x$
2. $\frac{d}{dx}(a^x) = a^x \ln a$
3. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
4. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

Trigonometric Functions

1. $\frac{d}{dx}(\sin x) = \cos x$
2. $\frac{d}{dx}(\cos x) = -\sin x$
3. $\frac{d}{dx}(\tan x) = \sec^2 x$
4. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
5. $\frac{d}{dx}(\sec x) = \sec x \tan x$
6. $\frac{d}{dx}(\cot x) = -\csc^2 x$

Table of Integrals

1. $\int u\,dv = uv - \int v\,du$
2. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{du}{u} = \ln|u| + C$
4. $\int e^u du = e^u + C$
5. $\int \sin u\,du = -\cos u + C$
6. $\int \cos u\,du = \sin u + C$
7. $\int \sec^2 u\,du = \tan u + C$
8. $\int \csc^2 u\,du = -\cot u + C$
9. $\int \sec u \tan u\,du = \sec u + C$
10. $\int \csc u \cot u\,du = -\csc u + C$

Application of Integrals:

Areas between Curve, $A = \int_a^b [f(x) - g(x)]\,dx$

Differential Equations

Linear Differential Equations

$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x)\,dx$, where $\mu = e^{\int p(x)\,dx}$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2 - 4ac > 0$)	$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
Repeated Roots ($b^2 - 4ac = 0$)	$y = c_1 e^{rx} + c_2 x e^{rx}$
2 Complex Roots ($b^2 - 4ac < 0$)	$y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

Constant Coefficient of Non-Homogeneous Equations

$y = y_c + y_p$ [y_c: complementary solution, y_p: particular solution]

Vector

Length of Vector

The length of the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Dot Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$

Cross Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \times \mathbf{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

Area for parallelogram *PQRS*

$= \left|\vec{PQ} \times \vec{PR}\right|$

Area for triangle PQR

$= \frac{1}{2}\left|\vec{PQ} \times \vec{PR}\right|$

Equation of Lines

Vector equation: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$

Parametric equations: $x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$

Symmetric equation: $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$

Equation of Planes

Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Linear equation: $ax + by + cz + d = 0$

Angle between Two Planes: $\theta = \cos^{-1}\left(\frac{\mathbf{n}_1 \bullet \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}\right)$